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When $t=-3$, $\quad x=0$, or $\pm a$.

$y=0$, or $\mp 3b$.

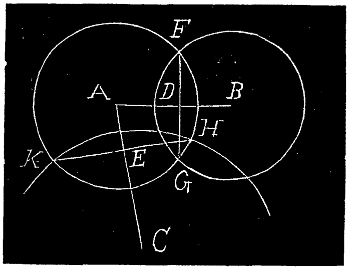
Also solved by *J. M. BOORMAN*, and *HARRY S. VANDIVER*.

GEOMETRY.

143. Proposed by J. T. FAIRCHILD, A. M., Instructor in Mathematics, Crawfis College, Crawfis College, Ohio.

If the centers of three spheres do not lie in the same straight line, their surfaces cannot have more than two points in common. These points lie in a straight line perpendicular to the plane of centers and equal distances from this plane on opposite sides. [From *Phillips and Fisher's Geometry*.]

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.



Let A, B, C be the centers of the three spheres, respectively. B intersects A in the circle, radius FD, such that FD is perpendicular to AB and D lies on AB. C intersects A in the circle, radius EH, such that EH is perpendicular to AC and E lies on AC. The two circles D and E on the same sphere A can intersect in only two points situated on a line perpendicular to the plane AED and equally distant from it. But plane AED coincides with plane ABC. Therefore, the truth of the theorem follows.

144. Proposed by L. C. WALKER, Assistant in Mathematics, Leland Stanford, Jr. University, Palo Alto, Cal.

Find the equations of four cones that pass through three given straight lines intersecting in the same point.

I. Solution by the PROPOSER.

Let the mutual inclination of the line be 2α, 2β, 2γ, and let the equation of the cone be referred to the three given straight lines as coördinate axes.

The equation of the concentric sphere referred to the same axes is

$$x^2+y^2+z^2+2yz\cos2\alpha+2zx\cos2\beta+2xy\cos2\gamma=r \quad \ldots(1).$$

The equation of the plane that passes through the intersection of the cone and (1) is

$$\frac{x}{r}+\frac{y}{r}+\frac{z}{r}=1\ldots.(2).$$

Now making (1) homogeneous by means of (2), and reducing, we have for the equation of the cone,

$$\frac{\sin^2\alpha}{x}+\frac{\sin^2\beta}{y}+\frac{\sin^2\gamma}{z}=0.$$

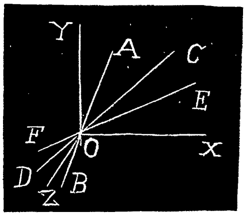
By using supplementary angles, the other equations are

$$\frac{\sin^2\alpha}{x}+\frac{\cos^2\beta}{y}+\frac{\cos^2\gamma}{z}=0,$$

$$-\frac{\cos^2\alpha}{x}+\frac{\sin^2\beta}{y}+\frac{\cos^2\gamma}{z}=0, \text{ and } -\frac{\cos^2\alpha}{x}-\frac{\cos^2\beta}{y}+\frac{\sin^2\gamma}{z}=0.$$

II. Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let O, the intersection of the three lines, be the origin of rectangular coördinates, and AB, CD, EF the three lines,



$$\left.\begin{matrix} x=az \\ y=bz \end{matrix}\right\}\ldots.(1),\ \left.\begin{matrix} x=cz \\ y=dz \end{matrix}\right\}\ldots.(2),\ \left.\begin{matrix} x=fz \\ y=gz \end{matrix}\right\}\ldots.(3),$$

the equations of AB, CD, DE, respectively, and

$$\left.\begin{matrix} x=tz \\ y=uz \end{matrix}\right\}\ldots.(4),$$

the equation of the axes of the cone having AO, BO, CO in the surface of one nappe. Then

$$\left.\begin{matrix} x=tz \\ y=-uz \end{matrix}\right\}\ldots.(5),\ \left.\begin{matrix} x=-tz \\ y=uz \end{matrix}\right\}\ldots.(6),\ \left.\begin{matrix} x=-tz \\ y=-uz \end{matrix}\right\}\ldots.(7),$$

are the equations of the axes of the remaining three cones.

$$\therefore\ (x-tz)^2+(y-uz)^2=e^2z^2(1+t^2+u^2)\ldots.(8),$$

$$(x-tz)^2+(y+uz)^2=e_1^2z^2(1+t^2+u^2)\ldots.(9),$$

$$(x+tz)^2+(y-uz)^2=e_2^2z^2(1+t^2+u^2)\ldots.(10),$$

$$(x+tz)^2+(y+uz)^2=e_3^2z^2(1+t^2+u^2)\ldots.(11),$$

are the equations of the cone.

Since (4) must make equal angles with (1), (2), (3), we get at once, if θ is this angle,

$$\cos\theta=\frac{1+at+bu}{\sqrt{[1+a^2+b^2]}\sqrt{[1+t^2+u^2]}}=\frac{1+ct+du}{\sqrt{[1+c^2+d^2]}\sqrt{[1+t^2+u^2]}}$$

$$=\frac{1+ft+gu}{\sqrt{[1+f^2+g^2]}\sqrt{[1+t^2+u^2]}}.$$

$$\therefore \frac{1+at+bu}{\sqrt{[1+a^2+b^2]}}=\frac{1+ct+du}{\sqrt{[1+c^2+d^2]}}=\frac{1+ft+gu}{\sqrt{[1+f^2+g^2]}} \text{ determines } t \text{ and } u.$$

$$e^2=\tan^2\theta=\frac{(t-a)^2+(u-b)^2+(au-bt)^2}{(1+at+bu)^2},\ e_1^2=\frac{(t-a)^2+(u+b)^2+(au+bt)^2}{(1+at-bu)^2},$$

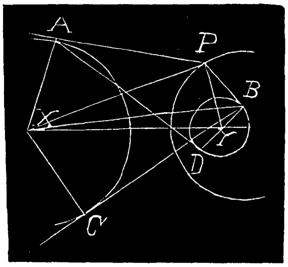
$$e_2^2=\frac{(t+a)^2+(u-b)^2+(au+bt)^2}{(1-at+bu)^2},\ e_3^2=\frac{(t+a)^2+(u+b)^2+(au-bt)^2}{(1-at-bu)^2}.$$

145. Proposed by FRANK GIFFIN, Graduate Student, State University, Boulder, Col.

If A and B be the points of contact, upon two circles X and Y, of tangents drawn from any point of their circle of similitude, then the tangent from A to Y is equal to the tangent from B to X. [From *Casey's Sequel to Euclid*, Part I., page 144.]

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let P be any point on the circle of similitude, AP, BP the tangents from P to X and Y, respectively. AD, BC the tangents from A and B to Y and X, respectively.



Let $AX=R$, $BY=r$. Let $AD=a$, $BC=b$, $AP=c$, $BP=d$, $PX=m$, $PY=n$.

$\angle APX=\angle BPY$ since P is on circle of similitude. $\therefore\ \angle APY=\angle BPX=\theta$.

Also, $c:d=R:r$. $\therefore\ d=cr/R$....(1).

$m:n=R:r$. $\therefore\ n=mr/R$....(2).

$a^2=AY^2-r^2=c^2+n^2-2cn\cos\theta-r^2$

$=c^2+m^2r^2/R^2-(2cmr/R)\cos\theta-r^2$....(3).

$b^2=BX^2-R^2=d^2+m^2-2dm\cos\theta-R^2=c^2r^2/R^2+m^2-(2cmr/R)\cos\theta-R^2$..(4).

(3)—(4) gives $R^2(a^2-b^2)=(c^2-m^2+R^2)(R^2-r^2)$.

But $c^2+R^2=m^2$.

$\therefore\ a^2-b^2=0$. $\therefore\ a=b$.

CALCULUS.

106. Proposed by M. C. STEVENS, M. A., Professor of Higher Mathematics, Purdue University, Lafayette, Ind.

$$\int_0^\pi \frac{\cos rx dx}{1-2a\cos x+a^2}=\frac{\pi a^r}{1-a^2}.$$

[*Williamson's Integral Calculus*, 6th Edition, page 174.]

Solution by WILLIAM HOOVER, A.M., Ph.D., Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio.

In Todhunter's *Plane Trigonometry*, 3d Edition, Art. 309, we have

$$\cos\alpha+a\cos(\alpha+\beta)+a^2\cos(\alpha+2\beta)+\ldots\ a^{n-1}\cos[\alpha+(n-1)\beta]$$